10.1.14

Domain: we need $9-x^2-y^2 \geq 0$, which implies that the domain is $\{(x,y) \mid x^2+y^2 \leq 9\}$.

Range: One should be able to see that $0 \leq 9-x^2-y^2 \leq 9$. Thus the range of $z=f(x,y)$ is $\{z \mid 0 \leq z \leq 3\}$

Level curves: Fix C, $0 \leq C \leq 3$, we have

$C = \sqrt{9-x^2-y^2}$, so that $x^2+y^2 = 9-C^2$, which is a circle of radius $\sqrt{9-C^2}$.

10.1.16

Domain: Since exp is defined everywhere, domain is $\mathbb{R}^2$.

Range: Since $-(x^2+y^2) \leq 0$ and exp is increasing, range is $\{0 < z \leq 1\}$. (0 is not included since exp is non-zero).

Level curves:

Fix C, $0 < C \leq 1$, we have $C = \exp(-(x^2+y^2))$, so that $-(x^2+y^2) = \ln C$, and $x^2+y^2 = \ln\frac{1}{C}$, which is a circle w/ radius $\sqrt{\ln\frac{1}{C}}$.

10.2.2

Since $2xy+3x^2$ is continuous at $(-1,1)$, we have

$$\lim_{(x,y)\to(-1,1)} 2xy+3x^2 = 2(-1)(1)+3(1)^2 = 1$$

10.2.12

Since $2xy+2$ is nonzero at $(-1,-2)$, so the rational function $\frac{x^2-y^2}{2xy+2}$ is continuous at $(-1,-2)$.

$$\lim_{(x,y)\to(-1,-2)} \frac{x^2-y^2}{2xy+2} = \frac{(-1)^2-(-2)^2}{2(-1)(-2)+2} = \frac{-3}{6} = -\frac{1}{2}$$

10.2. 16

Along positive x-axis

$$\lim_{\substack{x\to 0^+\\ y=0}} \frac{3x^2-y^2}{x^2+y^2} = \lim_{x\to 0^+} \frac{3x^2}{x^2} = 3$$

Along positive y-axis

$$\lim_{\substack{x=0\\ y\to 0^+}} \frac{3x^2-y^2}{y^2+y^2} = \lim_{y\to 0^+} \frac{-y^2}{y^2} = -1$$

Since the two limits does not agree, $\lim_{(x,y)\to(0,0)} \frac{3x^2-y^2}{x^2+y^2}$ DNE

$\boxed{\text{Remark on 10.2.2 and 10.2.12}}$

You can also apply the properties of Limit to those two problems i.e., Limit Laws on page 513. For example,

$$\lim_{(x,y)\to(1,-1)} 2xy+3x^2 = \lim_{(x,y)\to(1,-1)} 2xy + \lim_{(x,y)\to(1,-1)} 3x^2 = 2\left(\lim_{(x,y)\to(1,-1)} x\right)\left(\lim_{(x,y)\to(1,-1)} y\right) + 3\lim_{(x,y)\to(1,-1)} x^2$$

$$\lim_{(x,y)\to(-1,-2)} \frac{x^2-y^2}{2xy+2} = \frac{\lim_{(x,y)\to(-1,-2)} x^2-y^2}{\lim_{(x,y)\to(-1,-2)} 2xy+2}$$